

Вариант 6

№ 1

$$df = (0001010001000011)$$

| $x_1$ | $x_2$ | $x_3$ | $x_4$ |   |
|-------|-------|-------|-------|---|
| 0     | 0     | 0     | 0     | 0 |
| 0     | 0     | 0     | 1     | 0 |
| 0     | 0     | 1     | 0     | 0 |
| 0     | 0     | 1     | 1     | 1 |
| 0     | 1     | 0     | 0     | 0 |
| 0     | 1     | 0     | 1     | 1 |
| 0     | 1     | 1     | 0     | 0 |
| 0     | 1     | 1     | 1     | 0 |
| 1     | 0     | 0     | 0     | 0 |
| 1     | 0     | 0     | 1     | 1 |
| 1     | 0     | 1     | 0     | 0 |
| 1     | 0     | 1     | 1     | 0 |
| 1     | 1     | 0     | 0     | 0 |
| 1     | 1     | 0     | 1     | 0 |
| 1     | 1     | 1     | 0     | 1 |
| 1     | 1     | 1     | 1     | 1 |

1) СДНФ:  $\bar{x}_1 \bar{x}_2 x_3 x_4 \vee \bar{x}_1 \bar{x}_2 \bar{x}_3 x_4 \vee x_1 \bar{x}_2 \bar{x}_3 x_4 \vee$

$\vee x_1 x_2 x_3 \bar{x}_4 \vee x_1 x_2 x_3 x_4$

2) Пошлюк Мегашкина (булевой способ):

$$00|01|01|00|01|00|00|01|11$$

$$00|01|01|00|01|00|00|01|10$$

$$(00|01|01|01|01|01|01|01|10$$

$$(00|01|01|00|00|01|01|01|11$$

$$00|01|01|00|00|01|00|00|11$$

$$0011$$

$$0101$$

$$1001$$

$$1110$$

$$1111$$

Пошлюк Мегашкина:  $x_3 x_4 \oplus x_2 x_4 \oplus x_1 x_4 \oplus x_1 x_2 x_3 \oplus x_1 x_2 x_3 x_4$



$$N=2$$

Доопр. до линейной.

$$df = (--- 0 - 1 - 1 --- 1 ---)$$

$$\begin{matrix} \uparrow & \uparrow & \uparrow & \uparrow \\ 0011 & 0101 & 0111 & 1100 \end{matrix}$$

линейная ф-я имеет вид:

$$f = c_0 \oplus c_1 x_1 \oplus c_2 x_2 \oplus c_3 x_3 \oplus c_4 x_4$$

Используя известные значения составим систему:

$$\begin{cases} c_0 \oplus c_3 \oplus c_4 = 0 \\ c_0 \oplus c_2 \oplus c_4 = 1 \\ c_0 \oplus c_2 \oplus c_3 \oplus c_4 = 1 \\ c_0 \oplus c_1 \oplus c_2 = 1 \end{cases} \Leftrightarrow \begin{cases} \cancel{c_2 = 1} \\ 0 \oplus c_2 = 1 \\ 1 \oplus c_3 = 1 \\ c_0 \oplus c_2 \oplus c_3 \oplus c_4 = 1 \\ c_0 \oplus c_1 \oplus c_2 = 1 \end{cases}$$

$$\begin{cases} c_2 = 1 \\ c_3 = 0 \\ c_0 \oplus 1 \oplus 0 \oplus c_4 = 1 \\ c_0 \oplus c_1 \oplus 1 = 1 \end{cases} \Leftrightarrow \begin{cases} c_2 = 1 \\ c_3 = 0 \\ c_0 \oplus c_1 = 0 \\ c_0 \oplus c_4 = 0 \end{cases}$$

Рассмотрим <sup>возможных</sup> 2 случая:  $c_0 = 1, c_1 = 1, c_4 = 1$

и  $c_0 = 0, c_1 = 0, c_4 = 0$

$$\textcircled{1} c_0 = 1, c_1 = 1, c_2 = 1, c_3 = 0, c_4 = 1$$

$$f_1 = 1 \oplus x_1 \oplus x_2 \oplus x_4 \Rightarrow df = (1010 \ 0101 \ 0101 \ 1010)$$

| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $f_1$ | $f_2$ |
|-------|-------|-------|-------|-------|-------|
| 0     | 0     | 0     | 0     | 1     | 0     |
| 0     | 0     | 0     | 1     | 0     | 0     |
| 0     | 0     | 1     | 0     | 1     | 0     |
| 0     | 0     | 1     | 1     | 0     | 0     |
| 0     | 1     | 0     | 0     | 0     | 1     |
| 0     | 1     | 0     | 1     | 1     | 1     |
| 0     | 1     | 1     | 0     | 1     | 1     |
| 0     | 1     | 1     | 1     | 0     | 1     |
| 1     | 0     | 0     | 0     | 0     | 0     |
| 1     | 0     | 0     | 1     | 1     | 0     |
| 1     | 0     | 1     | 0     | 0     | 0     |
| 1     | 0     | 1     | 1     | 1     | 0     |
| 1     | 1     | 0     | 0     | 1     | 1     |
| 1     | 1     | 0     | 1     | 0     | 1     |
| 1     | 1     | 1     | 0     | 1     | 1     |
| 1     | 1     | 1     | 1     | 0     | 1     |

$$\textcircled{2} c_0 = 0, c_1 = 0, c_2 = 1, c_3 = 0, c_4 = 0$$

$$f_2 = x_2$$

$$df = (0000 \ 1111 \ 0000 \ 1111)$$

$$\text{Ответ: 1) } (1010 \ 0101 \ 0101 \ 1010)$$

$$2) (0000 \ 1111 \ 0000 \ 1111)$$



$N=4$

мет 2  
113 3

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$A = \{f_1, f_2, f_3\}$  - найти мет?

$f_1 = x_1 x_2 (x_1 \sim x_2)$

$d_{f_2} = (10010110)$

$f_3 = (x_1 \sim x_2) \rightarrow x_2 x_3$

Рассмотрим функцию от-гусеро.

1)  $f_1 = x_1 x_2 (x_1 \sim x_2)$

| $x_1$ | $x_2$ | $x_1 \sim x_2$ | $x_1 x_2$ | $x_1 x_2 (x_1 \sim x_2)$ |
|-------|-------|----------------|-----------|--------------------------|
| 0     | 0     | 1              | 0         | 0                        |
| 0     | 1     | 0              | 0         | 0                        |
| 1     | 0     | 0              | 0         | 0                        |
| 1     | 1     | 1              | 1         | 1                        |

$$\begin{array}{ccc|ccc} 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{array}$$

$f_1 \in T_0, f_1 \in T_1, f_1 \notin S, f_1 \in M, f_1 \notin L$

2)  $d_{f_2} = (10010110)$

$f_2 \notin T_0, f_2 \notin T_1,$

$f_2 \in S, f_2 \notin M$

$10010110$

$11010111$

$11100110$

$11101000 \Rightarrow f_2 \in L$

| $x_1$ | $x_2$ | $x_3$ | $f_2$ |
|-------|-------|-------|-------|
| 0     | 0     | 0     | 1     |
| 0     | 0     | 1     | 0     |
| 0     | 1     | 0     | 0     |
| 0     | 1     | 1     | 1     |
| 1     | 0     | 0     | 0     |
| 1     | 0     | 1     | 1     |
| 1     | 1     | 0     | 1     |
| 1     | 1     | 1     | 0     |

3)  $f_3 = (x_1 \sim x_2) \rightarrow x_2 x_3$

$f_3 \in T_0, f_3 \in T_1$

$f_3 \notin S, f_3 \notin M$

| $x_1$ | $x_2$ | $x_3$ | $x_1 \sim x_2$ | $x_2 x_3$ | $f_3$ |
|-------|-------|-------|----------------|-----------|-------|
| 0     | 0     | 0     | 1              | 0         | 0     |
| 0     | 0     | 1     | 1              | 0         | 0     |
| 0     | 1     | 0     | 0              | 0         | 1     |
| 0     | 1     | 1     | 0              | 1         | 1     |
| 1     | 0     | 0     | 0              | 0         | 1     |
| 1     | 0     | 1     | 0              | 0         | 1     |
| 1     | 1     | 0     | 1              | 0         | 0     |
| 1     | 1     | 1     | 1              | 1         | 1     |

$$\begin{array}{ccc|ccc} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}$$

$00101001 \Rightarrow f_3 \notin L$



4)

|       | $T_0$ | $T_1$ | $L$ | $S$ | $M$ |
|-------|-------|-------|-----|-----|-----|
| $f_1$ | +     | +     | -   | -   | +   |
| $f_2$ | -     | -     | +   | +   | -   |
| $f_3$ | +     | +     | -   | -   | -   |

A - полная система

A - базис  $P_2$ .

$N = 6$

$$A = \{ \bar{x}, x_1 x_2, x_1 \oplus x_2, x_1 \rightarrow x_2, x_1 x_2 \vee x_3 \}$$

1)  $\bar{x} \notin T_0, \notin T_1, \notin S, \in L, \notin M$

2)  $x_1 x_2$

$$\Delta f = (0001)$$

$$\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{array}$$

~~0 0 0 1~~

3)  $x_1 \oplus x_2$

$f_3 \in T_0, \notin T_1, \notin S, \notin M, \in L$

$$\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{array}$$

4)  $x_1 \rightarrow x_2$

$f_4 \notin T_0, \in T_1, \notin S, \notin M, \notin L$

$$\begin{array}{cc|c} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{array}$$

$$\begin{array}{cc|cc} 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{array}$$



№

Возможные базисы

смет 3  
из 3

5)  $X_1 X_2 \vee X_3$

| $x_1$ | $x_2$ | $x_3$ | $x_1 x_2$ | $f_5$ |
|-------|-------|-------|-----------|-------|
| 0     | 0     | 0     | 0         | 0     |
| 0     | 0     | 1     | 0         | 1     |
| 0     | 1     | 0     | 0         | 0     |
| 0     | 1     | 1     | 0         | 1     |
| 1     | 0     | 0     | 0         | 0     |
| 1     | 0     | 1     | 0         | 1     |
| 1     | 1     | 0     | 1         | 1     |
| 1     | 1     | 1     | 1         | 1     |

|   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 |

$\Rightarrow \neq L$

$f_5 \in T_0, \in T_1, \notin S, \in M, \notin L$

|       | $T_0$ | $T_1$ | $L$ | $S$ | $M$ |
|-------|-------|-------|-----|-----|-----|
| $f_1$ | -     | -     | +   | +   | -   |
| $f_2$ | +     | +     | -   | -   | +   |
| $f_3$ | +     | -     | +   | -   | -   |
| $f_4$ | -     | +     | -   | -   | -   |
| $f_5$ | +     | +     | -   | -   | +   |

$$f_1 = \bar{x}$$

$$f_2 = x_1 x_2$$

$$f_3 = x_1 \oplus x_2$$

$$f_4 = x_1 \rightarrow x_2$$

$$f_5 = x_1 x_2 \vee x_3$$

Возможные базисы:

$$\{\bar{x}, x_1 x_2\}, \{\bar{x}, x_1 x_2 \vee x_3\},$$

~~$$\{\bar{x}, x_1 x_2, x_1 \oplus x_2\}, \{\bar{x}, x_1 x_2, x_1 \rightarrow x_2\},$$~~

~~$$\{\bar{x}, x_1 x_2, x_1 x_2 \vee x_3\},$$~~

~~$$\{\bar{x}, x_1 x_2 \vee x_3, x_1 \oplus x_2\}$$~~

~~$$\{\bar{x}, x_1 \rightarrow x_2, x_1 x_2 \vee x_3\}$$~~

~~$$\{\bar{x}, x_1 x_2, x_1 \oplus x_2\}$$~~

и все

Ответ:  $\{\bar{x}, x_1 x_2\}, \{\bar{x}, x_1 x_2 \vee x_3\}$



$$N=3$$

$$A = (L \setminus T_0) \cup S$$

$$|L| = 2^{n+1}$$

$$|T_0| = 2^{2^n - 1}$$

$$|S| = 2^{2^n}$$

|         |   |                         |
|---------|---|-------------------------|
| 0 ... 0 | 1 | $\leftarrow \notin T_0$ |
|---------|---|-------------------------|

$$f = 1 \oplus c_1 x_1 \oplus c_2 x_2 \oplus \dots \oplus c_n x_n$$

$$|L \setminus T_0| = 2^n$$

$$|A| = |L \setminus T_0| + |S| - |(L \setminus T_0) \cap S|$$

|                     |         |   |                         |         |
|---------------------|---------|---|-------------------------|---------|
| $ L \setminus T_0 $ | 0 ... 0 | 1 | $\leftarrow \notin T_0$ | $\in L$ |
|                     | 1 ... 1 | 0 | $\leftarrow \notin T_0$ | $\in S$ |

$2^n - 2$

$$|L \cap S| = 2^{n-1}$$

$2^n$  вариантов

$$c_0 = 1$$

|               |          |          |
|---------------|----------|----------|
| $\frac{n}{2}$ | 0        | 0        |
| $\vdots$      | $\vdots$ | $\vdots$ |
| $\frac{n}{2}$ | 1        | 0        |
| $\vdots$      | $\vdots$ | $\vdots$ |
| $\frac{n}{2}$ | 1        | 0        |
| $\vdots$      | $\vdots$ | $\vdots$ |
| $\frac{n}{2}$ | 1        | 0        |

$n$ -четное  $n$ -четн.

по индукции

$$1) n=2$$

$$1 \oplus c_1 x_1 - 2 \text{ способа}$$

$$2) 1 \oplus c_1 x_1 \oplus c_2 x_2 - 2 \text{ способа}$$

$$n=3$$

$$1 \oplus c_1 x_1 \oplus c_2 x_2 \oplus c_3 x_3$$

$$\begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}$$

$$|(L \setminus T_0) \cap S| = \frac{2^n - 2}{2} = 2^{n-1} - 1$$

$$|A| = 2^n + 2^{2^n} - 2^{n-1} + 1 = 2^{n-1}(2-1) + 2^{2^n} + 1$$

$$= 2^{n-1} + 2^{2^n} + 1$$

5 - 4 -